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16/ENG03/033

Mechatronics

ENG 281

1. The parametric equations of a curve are as given in equations 1 and 2

$x = \cos t + t \sin t$

$y = \sin t - t \cos t$

in terms of t, determine.

i. an expression for the radius of curvature (R) and;

ii. expressions for the coordinates (h, k) of the centre of curvature.

Solution.

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

$x = \cos t + t \sin t$; $y = \sin t - t \cos t$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{dx}{dt} = -\sin t + (1 \cdot + \cos t + \sin t)$$

$$= -\sin t + t \cos t + \sin t = t \cos t$$

$$\frac{dy}{dt} = \cos t + (-t \cdot -\sin t + \cos t - 1)$$

$$= +\sin \cos t + t \sin t - \cos t = t \sin t$$

$$\frac{dy}{dx} = t \sin t - t \cos t = \frac{t \sin t}{t \cos t} = \frac{\sin t}{\cos t}$$

$= \tan t.$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx} = \frac{d}{dt}(\tan t) \times \frac{1}{t \cos t}$$

$$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{1}{t\cos t}$$

recall $\sec^2\theta = \frac{1}{\cos^2\theta}$

$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

then $R = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2} = \frac{(1+\tan^2 t)^{3/2}}{\frac{1}{t\cos^3 t}}$

recall $1+\tan^2\theta = \sec^2\theta$

$$\frac{(\sec^2 t)^{3/2}}{(t\cos^3 t)^{-1}} = \frac{\left(\sqrt{\sec^2 t}\right)^3}{(t\cos^3 t)^{-1}}$$

$= \sec^3 t \times t\cos^3 t$

where $\sec\theta = \frac{1}{\cos\theta}$ $= \frac{1}{\cancel{\cos^3} t} \times t\,\cancel{\cos^3} t$

$R = t$

$\therefore$ $h = x_1 - R\sin t$

from $x = \cos t + t\sin t$

$h = \cos t + t\sin t - R\sin t$

$= \cos t + t\sin t - t\sin t$

$= \cos t$

$k = y_1 + R\cos t$

from $y = \sin t - \cos t$ $\quad R = t$

$k = \sin t - t\cos t + R\cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$